Prof. Hadl

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EDITED BY

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THE

MATHEMATICAL MONTHLY.

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PRIZE PROBLEMS FOR STUDENTS.

I. FIND a point without two concentric circles, from which if two tangents be drawn to the circles the one shall be the double of the other.

II. Any whole number and its fifth power, when divided by 30, leave the same remainder; also, the number and its seventh power, when divided by 42, leave the same remainder. — Communicated by ASHER B. EVANS, Madison University, Hamilton, N. Y.

III. There are n straight lines making, with another fixed straight line, the angles $\alpha, \beta, \gamma \ldots$; a point, P, is taken, such that the sum of the squares of the perpendiculars from it on these n lines is constant. Find the conditions that the locus of P may be a circle. — Communicated by GEORGE B. HICKS, Cleveland, Ohio.

IV. If A and B represent the semi-axes of an ellipse, the altitudes of the minimum circumscribing isosceles triangles, having their vertices in the axes produced, are as $A : B$. — Communicated by JAMES F. ROBERSON, Grantsburgh, Indiana.

V. Prove that, of all circular sectors having the same perimeter, the one of greatest area is that in which the circular arc is double the radius.

Solutions of these problems must be received by April 1st, 1861.

REPORT OF THE JUDGES UPON THE SOLUTIONS OF THE PRIZE PROBLEMS IN No. I., Vol. III.

The first Prize is awarded to A. G. Barker, Waterville College, Me.

The second prize is awarded to H. Teiman, Baltimore, Md.

The third prize is awarded to E. O. Gibson, South New Berlin, N. Y.

Prize Solution of Problem I.

By E. K. Leonard, Marietta College, Ohio.

Let AB and CD be two diameters of a given circle, drawn at right angles to each other; AEB a circular arc described with radius DA or DB. Prove that the area of the lune $AEBC$ equals the area of the triangle ADB.

Let R be the radius of the arc AEB, and r that of the circle. When $ABC = \frac{1}{2}\pi r^2$, $ADBC = \frac{1}{4}\pi R^2$, $r : R = 1 : \sqrt{2}$. Substituting 1 and $\sqrt{2}$ for r and R, we have $ABC = ADBE$. Subtracting ABE from each, we get $AEBC = ADB$.

Most of the solutions of this problem are nearly the same as the above.

Prize Solution of Problem II.

By E. O. Gibson, South New Berlin, N. Y.

Of all isoperimetrical polygons having the same number of sides, the greatest is that which is equilateral.

Since an isosceles triangle has a greater area than any other triangle of equal base and perimeter, it follows that if any polygon, $ABCD$, &c., be a maximum among others of equal perimeters and the same number of sides, all the triangles, ABC, BCD, &c., must be isosceles, and have $AB = BC = CD =$ &c.

Prize Solution of Problem III.

In any triangle, if P, P', and P'' denote the perpendiculars from the vertices to the opposite sides; p, p', and p'' the portions of the same between their point of intersection and the sides. Prove that

$$\frac{p}{P} + \frac{p'}{P'} + \frac{p''}{P''} = 1.$$

Mr. G. A. Osborne, Jr., says: Let a, b, c denote the sides of the triangle perpendicular respectively to P, P', P''. Then

$$a\,p + b\,p' + c\,p'' = 2 \text{ (area of triangle)},$$

and $$a\,P = b\,P' = c\,P'' = 2 \text{ (area of triangle)}.$$

Dividing each term of the first equation by the term below it, gives

$$\frac{p}{P} + \frac{p'}{P'} + \frac{p''}{P''} = 1.$$

Mr. Gibson notices that, in an obtuse-angled triangle, as the parts of the perpendiculars let fall on the sides produced, and which are represented by p and p', lie beyond those sides, they may be regarded as minus, and we still have

$$\frac{p}{P} + \frac{p'}{P'} + \frac{p''}{P''} = 1.$$

Mr. G. B. Hicks remarks, that this proposition may be included in the still more general statement, that, if from the angles of a triangle three lines, $A\,a$, $B\,b$, $C\,c$, be drawn to meet the opposite sides, and from any point, O, within the triangle three lines, $O\,\alpha$, $O\,\beta$, $O\,\gamma$, be drawn parallel to them to meet the sides, then

$$\frac{O\,\alpha}{A\,a} + \frac{O\,\beta}{B\,b} + \frac{O\,\gamma}{C\,c} = 1.$$

Prize Solution of Problem IV.

By A. G. Barker, Waterville College, Maine.

At what angle must the rudder of a vessel be inclined to the stream, that the effect produced may be a maximum?

Let the angle made by the rudder with the direction of the vessel be denoted by φ, the horizontal distance of the vessel's centre of gravity from the line of contact of the rudder by a, and the length of the rudder by b. The force exerted by the water upon the rudder parallel to the direction of the vessel's motion is proportional to $\sin\varphi$, since this force is zero for φ equal zero, and a maximum for φ

equal 90°. We may, therefore, take $\sin\varphi$ as its measure; and since φ is the angle this force makes with the rudder, its component perpendicular to the rudder is $\sin^2\varphi$, which we may suppose to act in the perpendicular passing through its middle point. This perpendicular produced will meet the line of direction of the vessel at a distance $\frac{\frac{1}{2}b}{\cos\varphi}$ from the line in which the rudder joins the vessel, and $a + \frac{\frac{1}{2}b}{\cos\varphi}$ is the length of the arm upon which the force $\sin^2\varphi$ acts at an angle $\frac{1}{2}\pi - \varphi$. The component of $\sin^2\varphi$ perpendicular to $a + \frac{\frac{1}{2}b}{\cos\varphi}$ is $\sin^2\varphi\cos\varphi$, and

$$u = \left(a + \frac{\frac{1}{2}b}{\cos\varphi}\right)\sin^2\varphi\cos\varphi = \max.$$

$$\begin{aligned}\therefore\ \frac{du}{d\varphi} &= 2a\sin\varphi\cos^2\varphi - a\sin^3\varphi + b\sin\varphi\cos\varphi = 0\\ &= 3a\cos^2\varphi + b\cos\varphi - a \qquad\qquad = 0;\end{aligned}$$

$$\therefore\ \cos\varphi = -\frac{b \pm \sqrt{(12a^2 + b^2)}}{6a}.$$

The second derivative gives

$$\frac{d^2u}{d\varphi^2} = -6a\cos\varphi\sin\varphi - b\sin\varphi,$$

which, for $\varphi < 180°$, is negative for the positive value of $\cos\varphi$; and therefore this value of $\cos\varphi$ will make u a maximum.

The negative value of $\cos\varphi$ gives

$$\frac{d^2u}{d\varphi^2} = \sqrt{(12a^2 + b^2)}\sin\varphi,$$

a positive quantity; and u is therefore a minimum for the negative value of $\cos\varphi$.

When $b = 0$, that is, when the length of the rudder is neglected, $\cos\varphi = \pm\sqrt{\frac{1}{3}}$; $\therefore\ \varphi = 54°\ 44'$, or $125°\ 16'$.

None of the solutions of Problem V. are thought worthy of publication.

Simon Newcomb,
W. P. G. Bartlett,
Truman Henry Safford.

DESCRIPTIVE GEOMETRY OF ONE PLANE.

CHAPTER I.

THE POINT, THE RIGHT LINE, AND THE PLANE.

The Point.

§ 1. A point in space is represented by its orthographic projection upon a fixed plane, called the *plane of reference*, and a number written near the projection, to show the distance of the point from this plane: this number is called the *reference* of the point with respect to the plane.

The plane of reference will at present be considered horizontal, and entirely below the magnitudes to be represented, thus avoiding any confusion which might arise from the use of negative references.

The Right Line.

§ 2. A right line in space is determined by the projections and corresponding references of two of its points.

§ 3. Problem I. *Given the projection of a point situated upon a given right line, to find the reference of the point.*

Let AB (Fig. 2) be the projection of a line, and let the references of two of its points, projected at A and B, be respectively α and β. It is required to find the reference of the point, of which C is the projection. If we imagine the line in space revolved about its horizontal projection as an axis, so as to coincide with the plane of reference, it will take the position $A'B'$, making AA' equal to α, and BB' equal to β. Calling the unknown reference of C, γ, we have

$$BA : BC = \alpha - \beta : \gamma - \beta,$$

whence
$$\gamma = \frac{BC}{BA}(\alpha - \beta) + \beta.$$

The lengths of AB and BC are known, having been measured in units of the scale (Fig. 1), and α and β are measured upon the same scale.

Conversely, if it be required to find the projection of that point of the line whose reference is γ, we have, from the same proportion,

$$BC = AB\left(\frac{\gamma - \beta}{\alpha - \beta}\right).$$

Or the point C' may be found by laying off on the line AA' a distance AD equal to γ, and drawing DC' parallel to AB.

§ 4. Prob. II. *To find the distance between two given points.*

Let A and B (Fig. 2) be the given points, then evidently the distance required will be $\sqrt{(AB^2 + (\alpha - \beta)^2)}$.

§ 5. *Definitions.* — The *tangent* of the angle made by the line with the plane of reference, $\left(\frac{\alpha - \beta}{AB}\right)$, is called the *declivity of the line*, and when put in the form $\frac{1}{m}$, it shows that points of the line whose projections are m units apart differ in reference by unity.

The distance between the projections of two points is called their *interval;* and the distance between the projections of two points of a line differing from each other by a unit of reference is called the *interval of the line.*

§ 6. By an application of the process explained in Problem I., it is easy to divide the projection of any given line into intervals; the projection thus divided is called the *scale of declivity of the given line.*

§ 7. Prob. III. *To construct the scale of declivity of a given line.*

Let the line be given by its projection, and the references 10.83 of the point A and 8.28 of B (Fig. 3); the interval of the points A and B, measured upon the scale (Fig. 1) is 5.75.

Therefore the declivity of the line is

$$\frac{10.83 - 8.28}{5.75} = \frac{2.55}{5.75} = \frac{1}{2.25} = \frac{1}{m}.$$

Therefore the interval of the line is $m = 2.25$. Therefore, setting off a distance 2.25 from A towards B, we have the point C, of which the reference is less by unity than that of A, i. e., 9.83.

But it is usual to divide the projection into integer references. To find the point 10, set off from A towards B a distance $2.25 \times 0.83 = 1.87$ of the scale (Fig. 1); from 10 set off a distance 2.25, which gives the point 9, &c. Divide one of the intervals, as 9 – 8, into 10 equal parts, and the scale will be completed.

§ 8. PROB. IV. *Through a given point to draw a line parallel to a given line.*

Let the line $a\,b$ be given by the projections and references of two points, 12 of a and 7.80 of b, and the point by its projection c and its reference 8.50. Draw through c a line parallel to $a\,b$, and lay off on it from c the interval $a\,b$; this gives a point d of the required line, of which the reference is $8.50 + 4.20 = 12.7$.

THE PLANE.

§ 9. The planes which are required to be represented are of two kinds, those which are limited in extent, as faces of polyhedrons, and those which are unlimited.

An unlimited plane is determined by the position of one of its lines of greatest declivity, that is, one of the lines of the plane making the maximum angle with the horizontal plane of reference; this line is evidently perpendicular to every horizontal line of the plane so defined.

The scale of declivity of this line is called the *scale of declivity of the plane,* and to distinguish it from that of an ordinary line it is made double.

A limited plane is represented by the projection of its perimeter, to which is added either its scale of declivity or the references of its angular points. Often a plane is given by two points of its line of greatest declivity.

§ 10. PROB. V. *To pass a plane through three given points.*

Let a, b, c (Fig. 5) be the points, of which the references are respectively 10.5, 7, and 4. Join a and c, and find on $a\,c$ the point d, whose reference is the same as b (7); join d and b. This will be a horizontal line of the required plane; the line of maximum declivity will be perpendicular to it.

To find another point, draw through c a line parallel to $d\,b$; it will be the horizontal (4), which determines the line of greatest declivity.

If we have the projection of any point, as e, situated upon this plane, to find its reference, it will be sufficient to drop a perpendicular from e to the scale, and find by Prob. I. the reference of the point of intersection, which will be the reference required.

§ 11. PROB. VI. *To find the intersection of two planes.*

Let the two planes M and N (Fig. 6) be given by their scales of declivity.

In one plane, M, draw any two horizontals; through 7 and 11, for instance. In the other plane, N, draw horizontals having the same references, and the points a and b, their intersections, are two points of the line of intersection of the two planes.

It may happen that the horizontals of each plane are nearly or quite parallel. In the first case, take two auxiliary planes, Q and P (Fig. 7). Q intersects M in the line $x\,y$, and also N in the line $v\,t$; therefore a, the intersection of $x\,y$ and $t\,v$, is a point common to the three planes M, N, and Q, or a point of the line of intersection of M and N. The same construction with P gives the point b; whence $a\,b$ is the line of intersection required.

If the horizontals were exactly parallel, one auxiliary plane would be sufficient, since the line of intersection would be a horizontal.

§ 12. PROB. VII. *Through a given point in a given plane, to draw a line of the plane having a given declivity.*

Let M (Fig. 8) be the given plane, a the projection of the given

point, and 2.7 its reference; let the declivity of the required line be $\frac{6}{10}$. Draw any horizontal of *M*, as 6, and take the difference between its reference and that of the given point $(6 - 2.7 = 3.3)$; multiply this difference by the reciprocal of the declivity of the required line $(3.3 \times \frac{10}{6} = \frac{11}{2} = 5.5)$; then with the projection of the given point as a centre, and a radius 5.5, describe an arc *c d*; and where it meets the horizontal 6 will be another point of the required line.

There will be two solutions, if the declivity of the line be less than that of the plane; one, if it be equal, none, if it be greater.

§ 13. Prob. VIII. *Through a given line pass a plane of a given declivity.*

Let the declivity of the required plane be $\frac{1}{m}$ (Fig. 9). Conceive any point of the line, as *b*, to be the vertex of a cone, of which the elements have a declivity $\frac{1}{m}$; to trace the base of the cone, it will be sufficient to take a radius, *m*, and with *b* as a centre describe a circle; the plane of this base will be horizontal, whose reference is less by unity than that of *b*. Now, from the point *c* upon the given line, whose reference is less than *b* by unity, draw a tangent to the circle; this tangent will be a horizontal of the required plane. Draw a parallel through *b*, and we have another, whence its line of declivity is immediately determined. In general there will be two solutions.

Sometimes it happens, in the solution of this problem, that the line being very little inclined, the position of the second point would lie off the paper. The solution for this case is to take any other point, *d*, of the line, and consider it also the vertex of a cone having its base on the same horizontal plane, and its declivity the same, the radius of its base would be *m* times the altitude; a horizontal of the plane would then be found by drawing a tangent common to the two circles of the base. (Fig. 10.)

If the line given were exactly horizontal, then one cone would be sufficient.

§ 14. PROB. IX. *To find the point in which a line pierces a plane.*

Let M (Fig. 11) be the given plane, and A the given line. Through the line A pass any arbitrary plane, X, and find its intersection, $m\,n$, with M; the point in which this line of intersection meets A is the point sought.

This method may be used to ascertain the point in which two lines having the same projection intersect.

Pass through the two given lines any two arbitrary planes, X and Y, and determine their intersection; the point in which this intersection meets the given lines is their common point. Or the point of intersection may be found by revolving the lines about their common projection as an axis until they are parallel to the horizontal plane of reference. [The projection may be considered as situated in any horizontal plane, since, as in this case, it is often convenient to work upon horizontal planes of considerable references.]

Fig. 12 represents these two solutions.

§ 15. PROB. X. *From a given point in space to drop a perpendicular upon a given plane, to find the point in which it pierces the plane and the length of the perpendicular.*

First Solution. — The projection of the perpendicular will evidently be parallel to the scale of declivity of the plane, and its declivity the reciprocal of that of the plane.

Suppose the declivity of the plane is $\frac{1}{m}$, then the interval upon the perpendicular will be $\frac{1}{m}$.

Therefore, through the projection of the given point a draw a line parallel to the scale of declivity of the plane, and lay off from a on this line intervals of $\frac{1}{m}$, measured upon the scale of equal parts (Fig. 1); the references of the perpendicular will increase in a direction contrary to that of the scale of declivity of the plane.

The point in which the perpendicular pierces the given plane may be found by § 16, and its length by § 4.

If the line of declivity of the plane be given by the references of two of its points, c and d, the perpendicular can still be drawn without constructing the scale of the plane. For after drawing the parallel to the line of declivity of the plane, set off on it from the point a a distance equal to the difference of the references of c and d, and the reference of this last point will be the reference of a *plus* or *minus* the length of the interval between c and d, according as it is laid off *above* or *below* the point a. That is, intervals of the plane correspond to differences of reference on the perpendicular, and reciprocally.

There is another solution to this problem, which may be used to advantage in many of the subsequent problems.

Let a (Fig. 13) be the given point, $d\,e$ a section of the plane of reference, $d\,c$ the line of maximum declivity of the given plane, $a\,c$ the required perpendicular, and $d\,e$ its horizontal projection; $\tan c\,d\,e = \frac{1}{m}$, the declivity of the given plane. Then the length of

$$a\,c = a\,d \cos \tan^{-1} \tfrac{1}{m}, \quad \text{and} \quad d\,e = a\,d \sin \tan^{-1} \tfrac{1}{m} \cos \tan^{-1} \tfrac{1}{m};$$

these two may be readily constructed in connection with the scale of declivity of the plane.

Let M (Fig. 14) be the given plane, and a be the projection of the point of which the reference is 11. Let fall from a a perpendicular to the scale, and find the corresponding reference 8.4 of b. From b set off a distance $b\,c$ equal to an interval of the scale M, and also set off $b\,d$ equal to a unit of the scale of equal parts (Fig. 1); join c and d, and produce this line. From c set off a distance $c\,e$ equal to 2.6, the difference of reference between b and a. Draw $e\,f$ perpendicular to $c\,d$, and $f\,h$ parallel to $c\,e$; and the point h, where the last line cuts the parallel through a to the scale M, is the point sought. $a\,h$ is the

projection of the required perpendicular and cf is its length; the angle fce is the angle of declivity of the plane.

The interval ah may be easily calculated. Let the declivity of the plane be $\frac{1}{m}$, and the difference of reference between the points a and b be denoted by δ; then the length of the required perpendicular is

$$p = \delta \cos \tan^{-1} \frac{1}{m} = \delta \cdot \frac{m}{\sqrt{(m^2+1)}},$$

$$ah = \delta \cdot \cos \tan^{-1} \frac{1}{m} \sin \tan^{-1} \frac{1}{m} = \delta \frac{m}{\sqrt{(m^2+1)}} \cdot \frac{1}{\sqrt{(m^2+1)}} = \delta \frac{m}{m^2+1}.$$

The difference of reference between the point h and the horizontal ab is $\delta \sin^2 \tan^{-1} \frac{1}{m} = \delta \frac{1}{1+m^2}$.

§ 16. PROB. XI. *To draw through a given point a plane perpendicular to a given line.*

The projection of the scale of declivity of the plane must be parallel to that of the given line, and its declivity is the reciprocal of that of the line.

So the scale is found in the same manner as that of the required line by the first method of the last problem.

§ 17. PROB. XII. *To find the shortest distance from a point, a, in space to a line, A.*

First Method. — Pass through a a plane perpendicular to A, and find the point in which A pierces this plane; the line connecting this last point with a will be the perpendicular required.

Second Method. — Let a (Fig. 15) be the given point, and A the given line; join A and the point b of A having the same reference (7), ab will be a horizontal of their plane (X); rotate this plane about ab as an axis until it becomes parallel to the horizontal plane of reference; any point of A, as c, will be found at c', by taking sc' equal to the distance between the axis and c in space. Therefore A' will be the revolved position of A; ad' will be the length of the

perpendicular, and, making the counter revolution, we have $a\,d$, its projection.

§ 18. Prob. XIII. *Through any two lines, A and B, pass two parallel planes.*

Through any point of A draw a line parallel to B; these two lines will determine the first plane. Then through any point of B draw a plane parallel to the first.*

§ 19. Prob. XIV. *Through a given line, A, to pass a plane perpendicular to a given plane, M.*

From any point of A drop a perpendicular upon M; this perpendicular and the line A determine the required plane.

§ 20. Prob. XV. *To measure the angle of two lines, A and B.*

Draw through any point of A (Fig. 17) a line parallel to B, as $a\,y$, and pass a plane through A and $a\,y$. Rotate this plane about any horizontal $x\,y$, so as to be parallel to the plane of reference. The new positions of the lines determine the angle sought, $x\,a'\,y$.

Since the three sides of the triangle $x\,a\,y$ are very easily determined, the angle may be found without rotation by the formula,

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,b\,c}.$$

§ 21. Prob. XVI. *Through a given point, a, to draw a line which shall make a given angle, μ, with a given line, A.*

Join a (Fig. 18) and the point of A having the same reference, this will be a horizontal of their common plane; rotate this plane till it is parallel to the plane of reference, and find A', the new position of the line A. Draw through a a line $a\,c'$, making the given angle μ with the line A', and make the counter revolution: $a\,c$ will be the line sought.

If the tangent of the angle μ be given, $\frac{1}{m}$, the point c' may be

* The construction for drawing one plane parallel to another is the same as for drawing one line parallel to another, and was explained by Prob. IV. Fig. 16 gives the construction.

found by dropping from a the perpendicular $a\,d'$, and measuring off the distance $d'\,c'$ equal to m times $a\,d'$.

§ 22. PROB. XVII. *To construct the angle of a given line, A, and a plane, M.*

From any point of A drop a perpendicular upon M, and by Prob. XV. find the angle between A and this perpendicular; this will be the complement of the angle sought.

§ 23. PROB. XVIII. *To find the scale of declivity of a line, knowing its projection, and also knowing it to be perpendicular to a given line, A, at a given point, a.*

Through the angular point a draw a plane, X, perpendicular to the given line. The unknown line will be in this plane; therefore drawing in X any horizontal, the point in which it meets the given projection of the unknown line will correspond to a point of the required line of the same reference; whence its scale may be immediately constructed.

§ 24. PROB. XIX. *To find the scale of declivity of a line, knowing its projection, and also the angle, μ, which it forms with a given line at a given point.*

If with the given angular point as a centre a sphere be conceived to be described, the four points of its surface in which the two lines and their two projections pierce it will form the angular points of a spherical quadrilateral, a, b, c, d (Fig. 19), in which $a\,b$ is the angle of declivity of the given line, $a\,d$ the given angle μ, $c\,b$ that of the projections, $a\,b\,c$, $b\,c\,d$ are right angles, and $d\,c$ is the unknown angle of declivity: whence the side $d\,c$ may be calculated by the rules of spherical trigonometry; but the graphic solution is as follows:—

Let a (Fig. 20) be the angular point, $a\,b$ the projection of the given line, $a\,e$ that of the unknown line. Pass a plane through the given line and the projection of the unknown line; its scale will be M, its angle of declivity $M\,f\,g$, the complement of which is $g\,f\,e$. Rotate

the line $a\,b$ about $a\,e$ as an axis, so as to coincide with the horizontal plane through a; it will take the position $a\,c$.

Make the angle $c\,a\,d$ equal to the given angle μ. Now, comparing Figs. 19 and 20, we have the three parts of the spherical triangle $a\,c\,d$ (Fig. 19), $a\,c$, $a\,c\,d$, and $a\,d$, which correspond respectively to $e\,a\,c$, $g\,f\,e$, and $c\,a\,d$ of (Fig. 20); and it is required to find the side $d\,c$ (Fig. 19), or, in other words, the other face of the pyramid. To do this, erect at any point m of $a\,e$ (Fig. 20) a perpendicular, $m\,n$; make the angle $m\,n\,r$ equal to $g\,f\,e$; then $m\,r$ is a line of the unknown face revolved about $n\,m$ as an axis, to coincide with the horizontal plane. At n erect a perpendicular to $m\,n$, and also draw $n\,v$ perpendicular to $a\,c$, and $n\,s$ perpendicular to $n\,v$ and equal to $n\,r$. The line $n\,v\,x$ is the horizontal trace of a vertical plane; and joining s and x, $s\,x$ is the revolved position of the intersection of the unknown face and the vertical plane $u\,x$. With v as a centre and $u\,v$ as radius describe an arc $u\,t$, cutting $s\,x$ in t. Let fall from t the perpendicular $t\,z$ to $u\,x$, and from z a perpendicular, $z\,y$, to $a\,e$. With x as a centre, and a radius $x\,t$, describe an arc meeting $z\,y$ produced in t'. Join t' and a, and we have $e\,a\,h$, the angle of declivity sought, and $a\,h$, the position of the line required revolved about its horizontal projection.

§ 25. PROB. XX. *To reduce an angle to the horizon; that is, having given μ the angle which a line forms with a given line at a certain point, and also its declivity, to find its projection.*

Let $a\,b$ (Fig. 21) be the given line, and a the angular point. Revolve the line about its projection so as to coincide with the horizontal plane through b; it will take the position $b\,a'$. Make the angle $a\,a'\,c$ equal to the angle of declivity of the unknown line, and $b\,a'\,d$ equal to μ, the given angle.

$a'\,e$ intersects $a\,b$ in c, therefore with a as a centre, and a radius $a\,c$, describe a circle; it will be the base of a cone of which a is the vertex, and every element of this cone will have the given declivity.

Take $a' d$ equal to $a' c$, and with b as a centre and $b d$ as a radius describe an arc intersecting the former in the points m and n; $a m$ or $a n$ will be the projection required. The line required and the given line are represented in revolved position by $b a' d$, and the third side of the triangle $b a' d$ is horizontal.

§ 26. *Transformation of the Plane of Reference.* — It is convenient often to change the plane of reference and refer the magnitudes to another plane whose scale with respect to the primitive is given; this may be done by an application of Prob. X.

I. *To change with respect to a point,* i. e. *to find the new projection and reference of a given point, a, with respect to any plane, M.*

Let fall from a a perpendicular upon M; find the point a', in which it pierces the plane M, and the length of the perpendicular.

The point a' is the new projection, and the length of the perpendicular is the reference sought.

II. *To change with respect to a line.*

Find the new projections and references of two points of the line. The construction may be simplified by taking as one point that in which the line pierces the plane M, which is its own projection, and has a reference zero.

III. *To change with respect to a plane; change with respect to any three points of the plane.*

The construction is much simplified by finding first the line of intersection of the given plane and the new plane of reference; this line is the new horizontal of zero;* dropping upon this intersection a perpendicular from the new projection of another point of the plane, this will be the projection of the new scale, which is known by the references of two points.

The scale of the primitive with respect to the new plane of refer-

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* The term horizontal here is evidently a misnomer, but it cannot lead to any ambiguity.

ence will have the same projection and interval, but will be the negative of it, *i. e.* it will increase from zero in the opposite direction.

The position of the new plane of projection may not be given directly, it may be required to be parallel or perpendicular to a given line or plane. The Problems IV., X., and XI. enable us to find immediately the scale of declivity required.

§ 27. Prob. XXI. *To find the angle of two planes, M and N.*

First Method. — Take the plane M as a new plane of reference; the new angle of declivity of N will be the angle sought. Fig. 22 gives the construction. Find the intersection $c\,e$ of the two planes M and N; this is the new horizontal. Let fall from any point of N, as a, a perpendicular upon M; it pierces it at a', and its length is $b\,d$. Let fall from a' a perpendicular, $a'f$, to $c\,e$, the new horizontal; then $a'f$ is the line of declivity of N with respect to M. Erect at a' a perpendicular to $a'f$ equal to $b\,d$, the new reference of a'; join g and f; $f\,g$ is the revolved position of the line of maximum declivity of N', and $a'f\,g$ is the angle required.

Second Method. — Let fall from any point in space, a, a perpendicular upon each plane, and measure the angle of these perpendiculars by § 20; this will be the supplement of the angle required. Or the length of the perpendicular upon either plane, as M, divided by the distance from its foot to the point in which the perpendicular upon N pierces the plane M, will be the tangent of the angle sought.

Third Method. — Let M and N (Fig. 23) be the two planes, $a\,b$ their line of intersection. Conceive a plane drawn perpendicular to $a\,b$ at the point y; $x\,y\,z$ will be a horizontal of this plane, having a reference 70. From y let fall a perpendicular, $y\,y'$, upon $a\,b'$, the revolved position of the line of intersection of the planes, and take $y\,y''$ equal to $y\,y'$, the angle $x\,y''\,z$ will be the angle sought.

§ 28. Prob. XXII. *Through a given right line to pass a plane which shall make a given angle, μ, with a given plane.*

Let A (Fig. 24) be the given line, and M the given plane. Take M as the new plane of reference.

To find the new position, A', of A, find first the point a, in which A pierces M; and then from any point of A, as d, let fall a perpendicular upon the plane M; it pierces this plane at the point c, and its length $e\,x$ is the new reference of c; therefore $a\,c$ is the new projection A'. Denote tan μ by $\frac{1}{m}$, the given declivity. Conceive any point of A', as k, the vertex of a cone, whose elements have the given declivity. The base of this cone is a circle situated in the new plane of reference, having for a centre k, and for a radius the reference of k multiplied by m.

Drawing from a a tangent to this circle, it will be a horizontal of the required plane with respect to the new plane of reference. This new horizontal meets the horizontal of zero of the primitive plane at the point y, therefore y is common to the primitive plane, to the new plane, and to the required plane; it is also its own projection in both planes, and has a reference zero. Therefore, joining y and the point z of the line A, whose reference is zero, we have the line $z\,y$, a horizontal of the required plane with respect to the primitive plane. Any other parallel, as through 40, is sufficient to determine the scale of declivity N of the required plane. The problem has in general two solutions. The other plane is determined by the other tangent, $a\,y'$, and has for a scale N'.

Second Method. — Let $\frac{1}{m}$ be the tangent of the given angle, A the given line, and M the given plane. From any point of A let fall a perpendicular upon M, and measure its length, p. Measure also the distance from the foot of p to the point in which A pierces M, and call this distance A'.

The distance from the foot of p to the intersection of the unknown plane with m is $m\,.\,p$, and the sine of the angle which A' makes with the same intersection is $\frac{m\,.\,p}{A'}$.

Through the foot of p pass a plane perpendicular to A'; and upon its intersection with M, starting from the foot of p, lay off a distance equal to $A' \times \tan\left(\sin^{-1}\frac{p\,.\,m}{A'}\right)$, or $\frac{A\,.\,p\,.\,m}{\sqrt{(A'^2 - p^2\,.\,m^2)}}$; and this point will be one of the required planes which, with the given line, immediately determines it.

§ 29. Prob. XXIII. *To determine the perpendicular between two right lines not in the same plane.*

First Solution. — Let A and B (Fig. 25) be the two lines. Pass a plane, X, perpendicular to A at any point, c, and take it as the new plane of reference. The new projection of B is B', and that of A is reduced to the point c. Draw $c\,r$ perpendicular to B'; this will be the perpendicular required. Since $c\,r$ in space is parallel to the plane X, the new reference of c (the point in which $c\,r$ meets the new projection of A) is the same as that of r. Therefore, returning to the primitive plane of reference, the point r is projected in t, and making $c\,s$ in space equal to the new reference of r, $s\,t$ will be the projection of the perpendicular between the lines A and B, and $c\,r$ will be its length.

Second Direct Solution. — Let A and B (Fig. 26) be the given lines. Pass through B a plane, X, parallel to A. Then from any point of A, as a, let fall a perpendicular to X; it pierces the plane X in a', and $c\,d$ is its length. A line through a parallel to A will be the projection of A upon the plane X. The point n, in which this projection meets B, is one point of the perpendicular sought; drawing $m\,n$ parallel to $a\,a'$, we have the required projection of the perpendicular distance between A and B.

§ 30. Prob. XXIV. *Given a plane, M, and the projection of a limited line, A, of this plane to construct upon A a cube which shall rest upon M.*

Let $a\,d$ (Fig. 27) be the length, A, of the given projection. Make the angle $s\,x\,y$ equal to the angle of declivity of M. Rotate the plane M about any horizontal x, as 50. To find the new position of

d, let fall from d a perpendicular to $s\,x$, and produce it to n on the line $x\,y$; then $x\,n$ will be the distance of d from the axis of rotation, and will make known the side $a\,d'$ of the cube. The square $a\,d'\,c'\,b'$ will be the base in revolved position, making the counter rotation, the base will be projected in $a\,b\,c\,d$. To find the upper base after revolution, make $x\,y$ equal $a\,d'$, and let fall the perpendicular $y\,s$; this will be the length of the horizontal projections of the edges of the cube projected at $a\,b\,c\,d$; whence, drawing through a, b, c, d lines perpendicular to the axis of rotation and equal to $y\,s$, we have the upper vertices; and the difference of reference between any two points symmetrically situated, as b and h, is $s\,x$, whence the cube is completely determined.

§ 31. PROB. XXV. *To find the intersection of two polyhedrons.*

Solution. — Pass a number of horizontal planes through each solid. Then the intersections of horizontals of the same reference, drawn in the intersecting adjacent faces of the two polyhedrons, make known the lines of intersection of those faces.

Fig. 28 represents an application of this method to the determinations of the intersections of two quadrangular pyramids.

§ 32. PROB. XXVI. *To find the shadow of any polyhedron.*

Let $a\,b\,c\,d\,e$ (Fig 29) be the lower base, and $a'\,b'\,c'\,d'\,e'$ the upper base of a polyhedron. Let the reference of the upper base be 80, and that of the lower base 0. Instead of supposing, as usual, the rays of light parallel to the diagonal of a cube resting upon the horizontal plane, let them be supposed to make an angle of 45° with the plane of reference, and have a direction indicated by the arrow A.

To find the shadow of any point, as f', draw through f' a parallel to A, and lay off a distance $f'f''$ equal to the reference 80 of f'. The same constructions for d' and e' give the points d'' and e''. Joining d and d'', f and f'', we have $f\,f''\,e''\,d''\,d$ as the shadow required.

NOTES AND QUERIES.

I. *A General Method of finding Tangents to Algebraic Curves.* — If the curve be referred to rectangular axes, and $x_1 y_1$ be the co-ordinates of the point of tangency, the equation of a line passing through this point will be of the form,

$$y - y_1 = a(x - x_1).$$

It is required to find the value of a when this line is tangent to the curve. Refer the curve to a system of polar co-ordinates, whose pole is at the point $x_1 y_1$, and axis parallel to the axis of x. Since the pole is upon the curve, the absolute terms will disappear from this equation. Every term of the equation will now contain r, the first power of which may be divided out. Then making $r = 0$, find the corresponding value of tan θ, which will evidently be the required value of a. This value of tan θ may always be found by the solution of an equation of the first degree. For any term that contains the product of sin θ and cos θ, or the higher powers of either, will also contain the higher powers of r, and will therefore disappear when $r = 0$. Hence it readily appears that in practice we may neglect all terms except those which contain the first power of r; and from these we shall find the required value of tan·θ.

This method will readily suggest its own demonstration.

Examples.

1. Take the folio of Descartes, of which the equation is

$$x^3 - 3cxy + y^3 = 0.$$

The equations of transformation are

$$x = x_1 + r\cos\theta, \qquad y = y_1 + r\sin\theta.$$

Hence we have

$$\left.\begin{array}{l} + x_1^3 + 3\,x_1^2\,r\cos\theta + \text{terms involving } r^2, \text{\&c.} \\ + y_1^3 + 3\,y_1^2\,r\sin\theta + \quad \text{“} \quad \text{“} \quad \text{“} \\ - 3\,c(x_1 y_1 + x_1\,r\sin\theta + y_1\,r\cos\theta + \text{terms involving } r^2, \text{\&c.} \end{array}\right\} = 0.$$

Striking out the terms independent of r, since

$$x_1^3 - 3\,c\,x_1 y_1 + y_1^3 = 0,$$

dividing by r, and then making $r = 0$, we have

$$3\,x_1^2\cos\theta + 3\,y_1^2\sin\theta - 3\,c\,x_1\sin\theta - 3\,c\,y_1\cos\theta = 0.$$

$$\therefore\ \tan\theta = \frac{c\,y_1 - x_1^2}{y_1^2 - c\,x_1};$$

and this result agrees with that obtained by differentiation.

2. The general equation of all parabolas is $y^m = a^{m-1}\,x$.

The transformed equation is

$$y_1^m + m\,y_1^{m-1}\,r\sin\theta + \text{\&c.} = a^{m-1}\,x_1 + a^{m-1}\,r\cos\theta.$$

Therefore, by neglecting the terms independent of r, dividing by r, and then making $r = 0$, we get

$$m\,y_1^{m-1}\sin\theta = a^{m-1}\cos\theta.$$

$$\therefore\ \tan\theta = \frac{a^{m-1}}{m\,y_1^{m-1}},$$

the true result.

This method may be extended to transcendental curves by means of algebraic expansion. — Walter Holliday, University of Virginia.

II. *Squares of Numbers ending in* 5, 25, *and* 75. — If in the square of the binomial $(t + u)^2 = (t^2 + 2\,t\,u + u^2)$, we put u equal to 5, or one half ten, it reduces to $(t^2 + t + \frac{1}{4}) = t\,(t + 1) + \frac{1}{4}$; but since the square of a number ending in 5 always ends in 25, we derive the following rule for squaring numbers ending with 5.

Square the unit's figure, annex to the left of its square the product of the ten's figure multiplied by itself plus unity.

Thus, $(25)^2 = 625$, $(65)^2 = 4225$, $(95)^2 = 9025$.

For numbers ending in 25 and 75 let h represent the hundreds, then 25 and 75 will be represented by $\frac{1}{4}$ and $\frac{3}{4}$; annexing each of these fractions to h, we have the two binomials $(h + \frac{1}{4})$ and $(h + \frac{3}{4})$; squaring each and factoring, they become

$$[h(h+5) + \tfrac{1}{16}], \qquad [(h+1)(h+5) + \tfrac{1}{16}],$$

each of which may be readily expressed in the form of a rule, by observing that the square of numbers ending in 25 or 75 always ends in 625. Thus,

$$(225)^2 = 50625, (725)^2 = 525625, (375)^2 = 140625, (975)^2 = 950625.$$

Let $h = 100$ and $t = 10$, then we have, for the powers of the above numbers,

$$\begin{aligned}
(225)^2 &= (\tfrac{9}{4}\,h)^2 = \tfrac{81}{16}\,h^2 = 50625, \\
(375)^2 &= (\tfrac{15}{4}\,h)^2 = \tfrac{225}{16}\,h^2 = 140625, \\
(225)^3 &= (\tfrac{9}{4}\,h)^3 = \tfrac{729}{64}\,h^3 = \tfrac{729}{8\times 8}\,h^3 = \tfrac{91125}{8} = 11390625, \\
(25)^4 &= (\tfrac{5}{2}\,t)^4 = \tfrac{625}{16}\,t^4 = 390625, \\
(15)^6 &= (\tfrac{3}{2}\,t)^6 = \tfrac{729}{64}\,t^6 = \tfrac{729}{8\times 8}\,t^6 = \tfrac{91125}{8} = 11390625.
\end{aligned}$$

Numbers may also be multiplied together in the same manner. Thus,

$$(15 \times 25 \times 5) = (\tfrac{3}{2}\,t \times \tfrac{1}{4}\,h \times \tfrac{1}{2}\,t) = \tfrac{3}{16}\,t^2 h = \tfrac{3}{16}\,h^2 = 1875,$$

$$(35 \times 75 \times 65 \times 5) = (\tfrac{7}{2}\,t \times \tfrac{3}{4}\,h \times \tfrac{13}{2}\,t \times \tfrac{1}{2}\,t) = \tfrac{273}{8\times 4}\,t^3 h = \tfrac{34125}{4} = 853125.$$

—James F. Roberson, State University, Bloomington, Indiana.

III. *Solution of Problem V., No. X., Vol. II.* — Suppose the sphere to be divided into an infinite number of concentric spherical shells; let A be their common centre, and B the exterior point; and M any particle on the surface of one of these shells, $r = AM$, $c = AB$, $y = BM$, $dr =$ the thickness of the shell, and $\rho =$ its density.

Since dr is infinitely small, ρ may be considered constant for the

same shell, but varying for *different* shells. The resultant attraction of the whole shell on

$$B = \frac{\pi \varrho r\, dr}{c^2} \int_{c-r}^{c+r} \left(1 + \frac{c^2 - r^2}{y^2}\right) dy = \frac{4 \pi \varrho r^2\, dr}{c^2}.$$

To obtain the part of the shell which exerts half this attraction, take $c - r$ and x for the limits of y, and put

$$(1) \qquad \frac{\pi \varrho r\, dr}{c^2} \int_{c-r}^{x} \left(1 + \frac{c^2 - r^2}{y^2}\right) dy = \frac{2 \pi \varrho r^2\, dr}{c^2}.$$

But $\int_{c-r}^{x} \left(1 + \frac{c^2 - r^2}{y^2}\right) dy = x - \frac{c^2 - r^2}{x} + 2r$; therefore equation (1) gives $x - \frac{c^2 - r^2}{x} + 2r = 2r$, or

$$(2) \qquad x = \sqrt{(c^2 - r^2)}.$$

If C be on the surface of the shell, and $BC = x$, the triangle ABC will be right angled at C. If we suppose r in equation (2) to take successively all values between zero and the radius of the sphere, the triangle ABC will be right angled in every case; hence the locus of the point C, which is always on the bisecting surface, is the surface of the sphere described on AB as diameter. — Asher B. Evans, Madison University, Hamilton, N. Y.

IV. *On an Approximate and Graphical Rectification of the Circle.** — The following treatment of this oft-attempted problem is founded upon the singularly close approach that is made by the angle $\tan^{-1}(\frac{1}{4}\pi)$ to a root of the equation $\sec x = \cot x$. The tabular solution of this equation is presented in the form

$$\cos x = \tan x = \sqrt{\left(\frac{\sqrt{(5)} - 1}{2}\right)} = 0.7863,$$

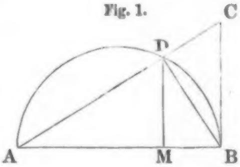
while $\frac{1}{4}\pi = 0.7854$; so that $x = 38° 10' 46''$, and $\tan^{-1} \frac{1}{4}\pi = 38° 8' 46''$:

* From the London Quarterly Journal of Pure and Applied Mathematics, October, 1860.

from which it is clear that if a simple construction can be found for obtaining the angle x that satisfies this equation, the tangent of that angle will yield a very close approximation to the desired periphery of a circle.

Let the chord ADC (Fig. 1) cut the semicircle ADB in D, and the tangent BC in C, in such a manner that $AD = BC$; then shall AD, BC represent *quam proximè* quadrants upon the semicircle ADB. By similar triangles, $\frac{AD}{AM} = \frac{AB}{AD}, = \frac{AB}{BC}$, by hypothesis. Or $\sec DAB = \cot DAB$, and DAB is such an angle as x; therefore

Fig. 1.



$$BC = AB \tan DAB = 2 \tan x, \text{ if the radius be unity;}$$
$$= 1.5727; \text{ and } \tfrac{1}{2}\pi = 1.5708.$$

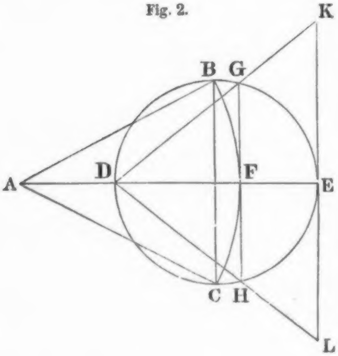
Therefore BC exceeds the quadrant of ADB by one thousandth part nearly; and in consequence, the triangle ABC exceeds the semicircle ADB by one thousandth part. It is upon such a singular coincidence as this that the following simple rule for approximate rectification of the circle with compass and ruler has been framed.

Rule.—"From an external point, at a distance equal to the radius from the given circle, describe a circle cutting the given circle at the two extremities of that diameter which is perpendicular to the line joining the point with the centre of the circle; and perpendicular to the same line draw two tangents to the two circles, the first touching the original circle at the point most remote from the external point, and the second touching the other circle, and cutting the original circle in two places. Then the two chords joining these two last points with the point of the given circle nearest to the external point, will represent the two quadrants that are there terminated; and if these chords be produced until they meet the tangent first

drawn, they will intercept upon it a portion bisected at the point of contact, and representing the two quadrants that are there terminated: and the triangle which this tangent forms with the two chords so produced will represent the area of the circle: and each of these rectilinear quantities will exceed the corresponding curvilinear quantity which it approximately represents by one thousandth part of the whole, nearly."

In proof of this construction, it will be seen that both DG and EK (Fig. 2) have by the construction the numerical values $\sqrt{[2\,\{\sqrt{(5)}-1\}]}$, radius being unity, and are therefore equal to one another, whence the inference proceeds as before.

Fig. 2.



This construction closely rivals in descriptive geometry the well-known arithmetical approximation to the value of π that is found in the vulgar fraction $\frac{22}{7}$, this fraction exceeding the true value by only four, and the above geometrical result exceeding the same by only twelve parts in ten thousand. This simple process will, for example, yield for the quadrant of a circle ten inches in diameter, a result exceeding the truth by less than the hundredth part of an inch, which amply suffices in accuracy for all that such processes can have in view.

P. S. — It has been pointed out to me that the equation $\sec x = \cot x$, which the angle $\tan^{-1} \frac{1}{4}\pi$ so nearly solves, is equivalent to that proposition which Herodotus has recorded concerning the dimensions of the great pyramid of Gizeh, when he says that the area of the sloping side is equal to the square upon the perpendicular height. The semiangle contained at the vertex between two opposite sloping sides is in this case such an angle as x, and

accordingly the above construction may be considered as affording a ready means for reproducing upon paper those proportions which a recent writer* has affirmed that this monument was intended to perpetuate, the periphery of its base bearing to its height very nearly the proportion of the circumference of a circle to its radius. The triangle AKL in the above construction will accordingly represent the vertical section of the Great Pyramid parallel to its sides.— A. S. HERSCHEL, Collingwood, England, August 18, 1860.

Mathematical Monthly Notices.

General Problems from the Orthographic Projections of Descriptive Geometry; with their Applications to Oblique — including Isometrical — Projections, Graphical Constructions in Spherical Trigonometry, Topographical Projection ("One Plane Descriptive"), and Graphic Transformations. By S. EDWARD WARREN, C. E., Professor of Descriptive Geometry and Geometrical Drawing in the Rensselaer Polytechnic Institute, Troy, N. Y. New York: John Wiley, 56 Walker Street, 1860. 8vo. pp. xxxv, 412.

The aims of PROFESSOR WARREN in this work are thus stated in his Preface: —

"1st. To proceed upon the basis of a perfectly accurate and rational idea of what Descriptive Geometry is.

"2d. To make the course perfectly *determinate* and rational, even to the very details, in the plan of its composition and the order of its progression.

"3d. To embody in its methods of treatment, as well as in its preparatory directions to the student, safeguards against false modes of study and sham scholarship.

"4th. To make the course sufficiently full to meet the wants of advanced students, and of the members of the professions based on mathematics and the physico-mathematical sciences."

The plan of the course is founded on the division of Surfaces into Ruled Surfaces and Double-curved Surfaces, and the subdivision of Ruled Surfaces into Plane Surfaces, Single-curved Surfaces, and Warped Surfaces. Upon each of these four classes of surfaces four operations are distinguished, namely, Projections, Intersections, Tangencies, and Developments.

In carrying out this plan the *General Problems* are placed under the two heads of *Lower Descriptive Geometry* and *Higher Descriptive Geometry*. Under the first head are treated quite fully the various problems relating to the plane, straight line, and point; those relating to the cone and cylinder, including conic sections; those relating to the hyperboloid of one nappe, and the hyperbolic paraboloid; and lastly, those relating to the sphere, ellipsoid, elliptic paraboloid, and hyperboloid of two nappes. Under the second head we find the construction of various spirals, the cissoid, conchoid, involutes, cycloids, helix, and spherical epicycloid; also problems on the developable helicoid, warped oblique arch, oblique helicoid, right conoid, and annular torus.

* "The Great Pyramid, and why it was built," by JOHN TAYLOR.

Book II. gives the *Applications of Orthographic Projections.* These include Isometrical Projections, Conventional Oblique Projections, Spherical Trigonometry, Topographical Projections (a name which the author prefers to One-plane Descriptive Geometry), and Graphic Transformations, which include changing the planes of projection and rotating given magnitudes. A collection of Miscellaneous Problems, sixty-three in number, closes the work.

From the above it will be seen that PROFESSOR WARREN'S book is far more comprehensive than any work on Descriptive Geometry before offered to American, or, so far as we know, to English readers. As such it will doubtless be welcomed by the Professors in our Scientific Schools, and by many students to whom the extensive French works on this subject are inaccessible. But, besides its comprehensiveness, this book has many positive merits. The author appears to have studied his subject well, and to be master of its minutest details. He is without doubt familiar with the works of his predecessors; but the marks of independent thought are apparent on even a cursory examination. His experience as a teacher has enabled him to judge of the wants of students, and to make many useful suggestions in the shape of remarks. We find also several examples of extended discussions of problems, that show a praiseworthy love of thoroughness.

With this general recognition of the merits of the book before us, we pass to some more particular remarks upon a few points.

The true sphere of Descriptive Geometry is one of the topics discussed in the Preface, and the conclusion arrived at is, that Descriptive Geometry is "the *exact graphic art*, and not the science of geometry, and is distinctively, *graphically operative geometry*, or the *Geometry of Graphic Problems.*" In this the author agrees with CHASLES (LIOUVILLE'S *Journal*, 1847, p. 33), who also denies to Descriptive Geometry the name of a science. But there are many able supporters of the opposite view, among whom may be noticed MONGE, CARNOT, PONCELET, and OLIVIER. M. OLIVIER discusses the subject quite fully in the preface to his *Additions au Cours de Géométrie Descriptive*, and becomes quite indignant against those who deny the claims of Descriptive Geometry to be called a science as well as an art.

PROFESSOR WARREN has also some remarks on the use of models and pictorial diagrams in teaching Descriptive Geometry, and condemns their general use, on the ground that without them certain powers of the mind — attention, abstraction, and especially conception — are better cultivated. This objection would be more forcible, if these powers could be cultivated in no other way, or if there were not many persons to whom Descriptive Geometry would be very useful, if it could be brought within their grasp without long and to them painful study. The truth is that no real simplification of any science is to be dreaded, even when we regard study as a discipline of the mind. When fields once inaccessible to ordinary minds are annexed to the common domain, there will always be higher fields opening up to exercise and strengthen the loftiest faculties.

OLIVIER, with an experience of more than twenty years in teaching Descriptive Geometry, recommends the use of models in the strongest terms. The costly models bearing his name are well known; but he has also contrived a simple apparatus which any one can have made for himself, and which we shall here describe for the benefit of those who have not access to this author's works. Take two shallow boxes similar to the tables of a backgammon board, and connected like them by hinges. Fill up these boxes with sheets of cork, and attach a brace to hold one table at right angles to the other, when necessary. These are the planes of projection. Provide four sets of rods from one eighth to one fifth of an inch square, and of various lengths from four inches to twelve, or sixteen, or twenty four inches, or even longer, according to the size of the planes. These rods are to be pointed with needles, that they may be readily fixed in any position in the cork. They are to be painted, one set red, one red and white in

bands, one black, and one black and white in bands. The red rods are used to construct in space any system referred in position to the two planes of projection; the red and white rods to project the points of this system on each of the planes of projection; the black and white to project these projections on the ground line; and the black to represent the projections of the red lines in space. When by means of these rods the operations necessary for the solution of a problem have been completed, the red and the red-and-white rods are removed, and the vertical plane revolved on its hinges to a horizontal position. The work is then done. By this apparatus also all ruled surfaces, such as cones, cylinders, hyperboloids of one nappe, hyperbolic paraboloids, conoids, &c., may be exhibited.

The classification of subjects adopted by PROFESSOR WARREN will arrest the attention of every one. Few, we think, will approve it. System is in itself a great good; but here the dividing, subdividing, and dividing again, are carried so far, that one almost doubts whether the confusion incident to so many heads is not greater than that of no classification whatever. A few distinct heads, such as those generally adopted by writers on this subject, are all that are necessary, at least in a text-book.

The style is also frequently open to objection. The greatest simplicity, directness, and precision should characterize such a work. All long and wordy sentences, all parade and circumlocution, all vagueness and ambiguity, are serious hindrances to the student. As an example of prolixity (one of the worst perhaps), we make an extract from the chapter on Graphic Transformations.

"426. Beneath all the formal problems of Descriptive Geometry, and giving character to their mode of solution, is a tacitly recognized problem, founded in the failure of human inventive skill to keep pace with human knowledge, as seen in the incapacity, hitherto apparent, to make *available*, that is, *cheap* and *permanently reliable*, instruments for the *easy* and *exact* graphical construction, by a continuous movement, of any lines but right lines and circles, — other lines being usually determined by points which are afterward joined by hand, or by the use of the 'irregular curve' (ruler).

"427. In continuation of this reasoning, it appears: 1st. That any point revolves about a fixed axis in a circular arc; 2d. In order to secure an adequate, that is, an immediately available, representation of such an arc, it must appear circular or rectilinear in projection; 3d. For this purpose, the plane of the arc must be parallel or perpendicular to the plane of projection; and 4th. This being the case, the axis of the arc will be respectively perpendicular or parallel to the same plane of projection."

We understand by this simply, that, since straight lines and circles are more easily drawn than other lines, it is best to choose an axis of rotation either parallel or perpendicular to the plane of projection; because then the path of a rotating point will be in projection a straight line, or an arc of a circle.

On page 5 we find: "Any magnitude, except a point, which is irreducible to anything more simple, may be regarded as the result of the motion of some simpler magnitude, according to a certain law." Here it is twice implied that a point is a magnitude; while on page 1 it is said that, "Magnitudes are definite portions of *space*, having one, two, or three dimensions." A point is therefore a definite *portion of space*, and has at least one dimension, — probably three, since its length, breadth, and thickness are very likely equal.

On page 104 we find: — "All sections of an oblique cone with a circular base which are parallel to the base, are circles, whose diameter depends on their distance from the vertex. All other sections, whose planes cut all the elements, are ellipses, save the sub-contrary circular section, *EF*, Pl. VIII., Fig. 53, whose plane makes an angle with an element, as *VB*, equal to the angle made by the parallel plane, *CD*, with the opposite element, *VA*."

Here are two omissions. It should have been stated, that the angles with VB and VA are both to be taken on the side towards the vertex. Moreover, there is no intimation that the figure does not represent *any* section of the cone through opposite elements; while it is essential that it should represent the particular section made by a plane drawn through the axis perpendicular to the plane of the base.

In drawing tangents to curves, the author appears to have had in view the method of ROBERVAL. This method is thus stated by its author (*Mém. Acad. des Sciences*, VI. p. 22): —

"*Axiom.* The direction of the motion of a point which describes a curve is a tangent of the curve in each position of this point.

"*General Rule.* By the specific properties of the curve (which are given) examine the different motions which the describing point has at the place where you wish to draw the tangent: compound these motions into one, and draw the line of direction of the compound motion; you will have the tangent of the curve."

PROFESSOR WARREN says (p. 72): —

"The tangent to a curve is therefore the direction in which the generatrix, when at the point of tangency, would move were it to cease to generate a curve, and actually to move in the direction of its tendency for the instant.

"But the law of the generation of a curve is such that the tendency of the generatrix at any instant is the resultant of two component tendencies, while from any common school philosophy we know that such a resultant corresponds to the diagonal of a parallelogram, whose two adjacent sides express the relative values and directions of the component tendencies."

Then follow applications to the conic sections. On pages 284, 385 (the subject being higher curves) we find the method restated thus: —

"The tangent line to the curve, at any point, is the resultant direction of the motion of the generatrix, due to the two component motions of that generatrix.

. . . . .

"The usual case is that neither of these motions is uniform. In this case, the tangent at any point is the resultant of the two tendencies for that instant only. Hence to construct the tangent at any point, draw two straight lines through that point in the directions in which the generatrix tends to move, at that point, and lay off on these lines distances whose ratio shall equal the *ratio* of the *rates* at which the generatrix tends to move at the instant in the given directions. The diagonal of the parallelogram, formed on these distances as two of its sides, will be the required tangent.

"To furnish such momentary ratios as are necessary to the above solutions is one object of the higher calculus."

Now the principle of ROBERVAL'S method is acknowledged to be correct; but in applying it there are undoubtedly difficulties in determining what the components are. ROBERVAL himself, MONTUCLA, and even MONGE, in one case, are said by DUHAMEL (*Mém. Sav. Étrang.*, V. 1838, p 257) to have confounded the component velocities of the generatrix with the projections of the total velocity upon the directions of the components. He shows also, that this confusion will produce no actual error, when the velocities are equal, or when the directions of the motions are at right angles to each other. But when a curve is given by an equation between the distances of its points to two or more foci, these conditions do not in general exist, and the error becomes manifest. To illustrate, suppose we had the equation $ar + br_1 =$ *a constant*, r and r_1 being the distances from any point of the curve to two fixed points, and a and b any multipliers, and let ε and ε_1 be the angles made by the tangent to the curve at any point with r and r_1. Then it is shown by DUHAMEL and by SERRET (*Méthodes en Géométrie*, p. 53, *et seq.*) that

$$\frac{dr}{dr_1} = -\frac{b}{a} = \frac{\cos \varepsilon}{\cos \varepsilon_1};$$

and, consequently, as will be readily seen by drawing a figure, that if we lay off on r and r_1 any distances having the ratio $dr : dr_1$, and erect perpendiculars at the extremities of these distances, the intersection of the perpendiculars will give the direction of the required tangent. It is obvious that the direction will not be that of the diagonal of a parallelogram constructed on the distances laid off as above, unless either $dr = \pm dr_1$, or the angle between r and r_1 is a right angle.

When, therefore, this method is used, the mode of finding the ratio of the components should be distinctly laid down. We have seen that it is possible to fall into error, even when the Differential Calculus is relied on to furnish the ratios. At any rate, if we have to rely on the Calculus for this essential step, the method ceases to be an independent one, and we had better adopt at once the recognized methods of drawing tangents derived from Analytical Geometry and the Calculus.

Extended as these remarks have become, we have left unnoticed many points that deserve attention. Enough, however, has been said to call attention to the principal features of this work. While we have not hesitated to point out what we consider faults, we trust that we properly appreciate the labors of the author in giving to the English student so extended, and, in many respects, so useful a work.

The Lady's and Gentleman's Diary, or Poetical and Mathematical Almanack for the Year 1861. London: Price one shilling and six pence.

This is the 158th annual number of this interesting and important publication. It is impossible to estimate the influence this unpretending serial has exerted upon the progress of the mathematics, and especially upon the development of mathematical talent in England. Besides 1988 original problems with their solutions, embracing all departments of the science, the mathematical papers thus far published would make several most valuable volumes. In our next we shall give the problems proposed for the next year, in hopes that many of our younger readers will feel disposed to send solutions of them "to the Editor of the Lady's and Gentleman's Diary, Stationer's Hall, London," post-paid, before May 1st, 1861.

Elements of the Differential and Integral Calculus. By William Smyth, A. M., Professor of Mathematics in Bowdoin College. Second Edition. Portland: Published by Sanborn and Carter. 1859.

The luminous statement, given in the Preface, of the spirit and plan of the work, at once arrests the attention; and an examination shows that the author is not only master of his subject, but clearly comprehends the manner of its presentation for the purpose of instruction. We have no hesitation in saying that this is one of the very best text-books upon the subject we have ever seen. The infinitesimal method of Leibnitz is adopted as its basis, which the author considers less logical than Newton's Method of Limits or Lagrange's Method of Derived Functions, but better adapted to the purposes of instruction, both on account of its greater simplicity, and the facility with which it is applied to the solution of the ordinary problems demanding the calculus, and especially for all purposes of investigation. Differentials are regarded simply as auxiliary quantities, and those who are familiar with the views of Carnot (to whom our author acknowledges his indebtedness), as exhibited in his incomparable little work, entitled, *Reflexions sur la Métaphysique du Calcul Infinitésimal,* will at once understand the spirit of the work before us. The arrangement is excellent, as the following brief synopsis will show. I. First Principles. II. Derivation of the auxiliary quantities, or differentials. III. Application to problems in which the differentials can be elim-

inated by the ordinary processes of Algebra. IV. Integral Calculus, in which the auxiliaries, or differentials, are eliminated by reversing the process by which they were obtained. V. Application to problems of quadrature, cubature, rectification; surfaces of solids. VI. Successive differentials, or new auxiliaries. VII. Application of calculus to development of functions. VIII. Maxima and minima. IX. Theory of curves. X. Transcendental functions, their differentiation and integration. XI. Transcendental curves. XII. Processes of integration. XIII. Cycloid. XIV. Quadratures and cubatures continued. XV. Application to Mechanics. XVI. Equilibrium, Centre of Gravity, Hydrodynamics. XVII. Method of Variations. XVIII. Application of Calculus to Astronomy. XIX. Limits, Derived Functions. XX. Miscellaneous examples.

The clearness and simplicity with which the principles are developed, the interesting historical references, and the variety of the problems, all combine to make the work before us an excellent text-book, and as such we commend it to teachers and students, with the assurance that they will not be disappointed.

The methods of NEWTON and LAGRANGE are very properly given at the end of the work, because the student is then prepared to compare them with the method of LEIBNITZ without confusion.

Editorial Items.

WE are happy to lay before our readers the following report; and especially so, because all the Essays submitted are found of "sufficient merit to be worthy of publication." We shall print the Essays in the order named as rapidly as possible, and all in this volume of the Monthly.

REPORT OF THE COMMITTEE ON PRIZE ESSAYS.

The Committee on Prize Essays regret that their Report has been so long delayed. The protracted absence of one of their number and the engagements of the rest prevented an earlier decision. They have received four Essays: On the Conformation of the Earth; On Central Forces; On Projections; and On Spherical Conics. All of these they consider of "sufficient merit to be worthy of publication;" although, as the productions of students, they are not to be too severely criticised. The Committee award the following prizes: To the first Essay, in the order named, forty dollars; to the second, thirty dollars; to the third, twenty dollars; and to the fourth, ten dollars.

WILLIAM FERREL,
J. B. HENCK,
CHAUNCEY WRIGHT.

BOOKS RECEIVED. — General Problems from the Orthographic Projections of Descriptive Geometry, &c., by PROF. S. EDWARD WARREN. (See Notice.) For sale by Brown and Taggard, Boston. — Annual Report of the Board of Regents of the Smithsonian Institution, showing the operations, expenditures, and condition of the Institution for the year 1859. — On the Equation of Differences for an Equation of any Order, and in particular for the Equations of the Orders Two, Three, Four, and Five, by ARTHUR CAYLEY, F. R. S.; also a paper on Recent Terminology in the Mathematics. — Progressive Intellectual, Practical, and Higher Arithmetics; New Elementary and University Algebras; New Plane and Solid Geometry, with Plane and Spherical Trigonometry; Surveying and Navigation, by H. N. ROBINSON, LL. D., published by Ivison, Phinney, & Co., 48 and 50 Walker Street, New York.

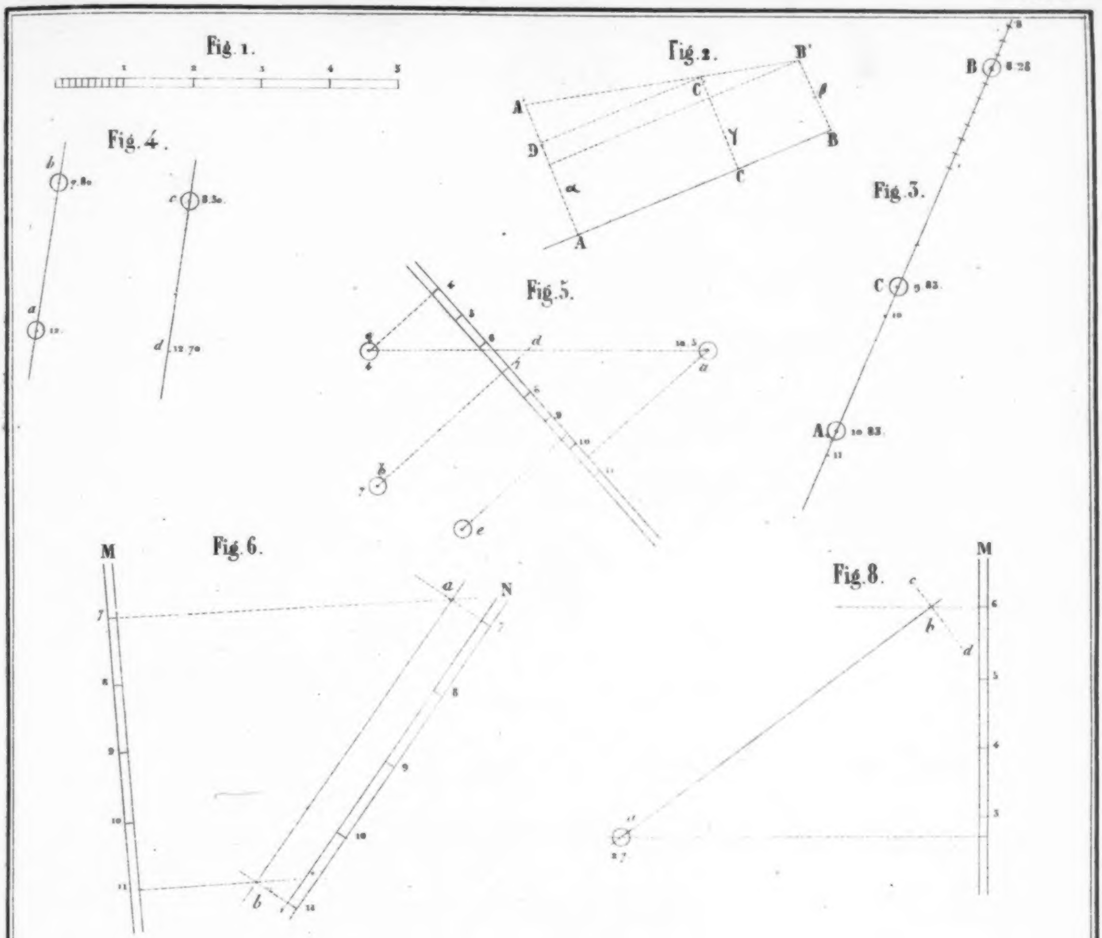


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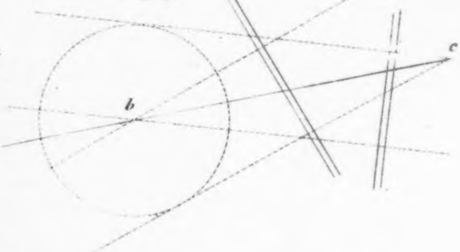
Fig. 9.



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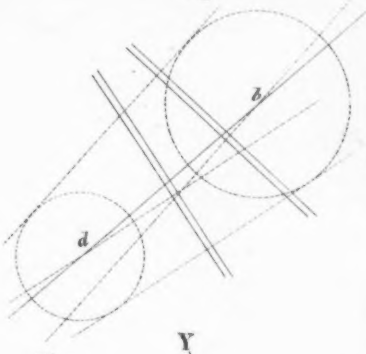

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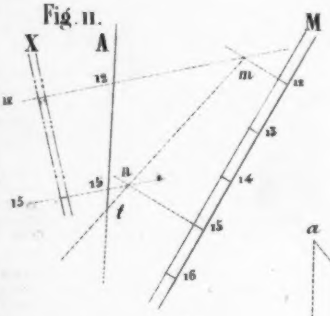

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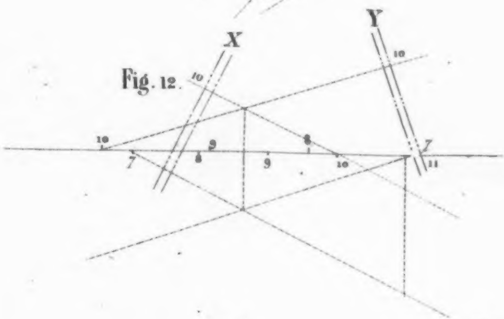

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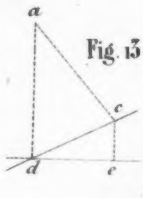

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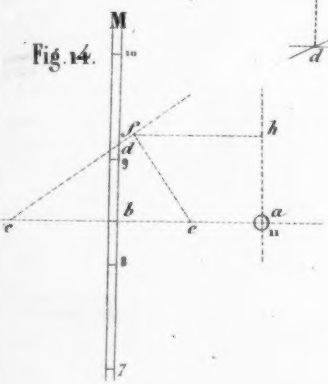

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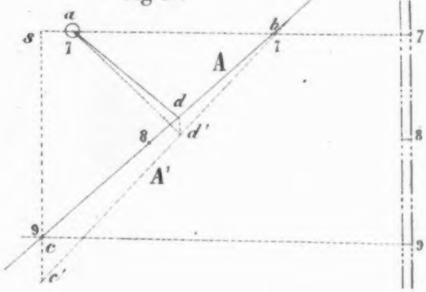

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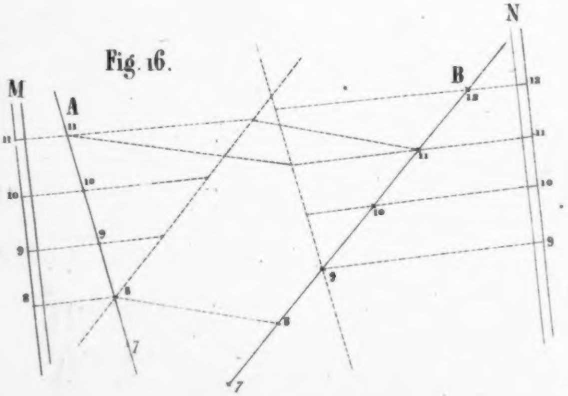

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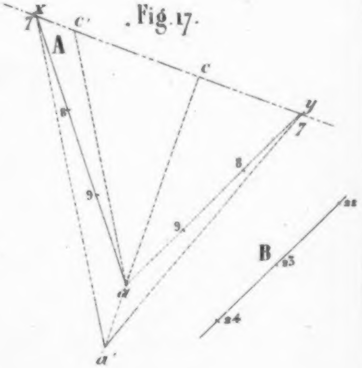

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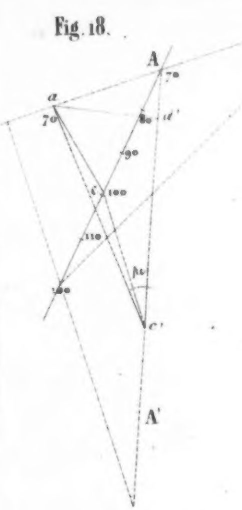

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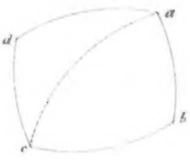

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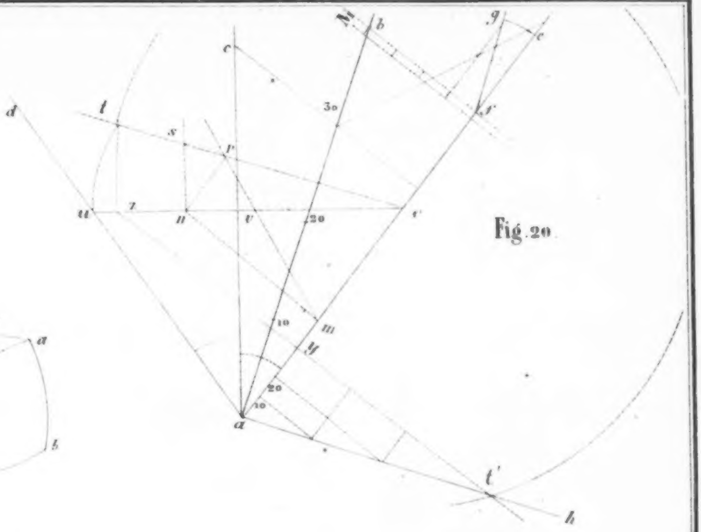

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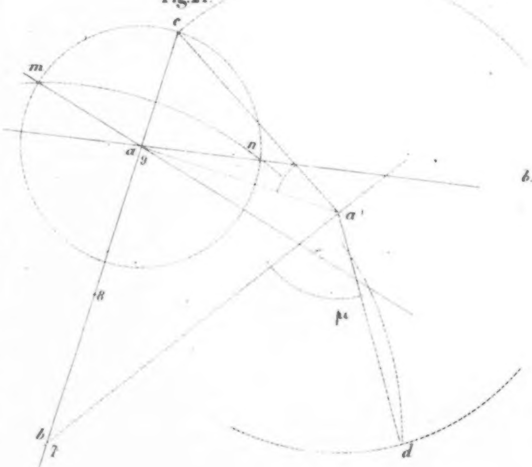

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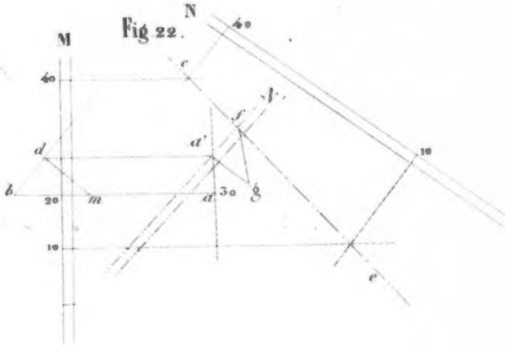

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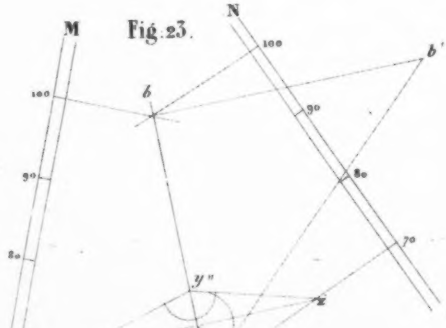

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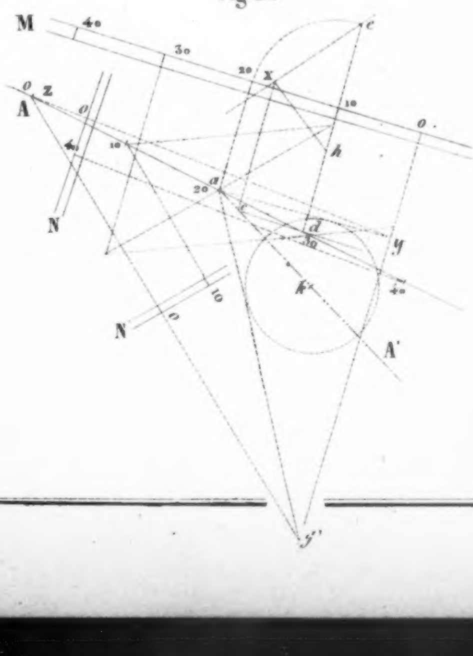

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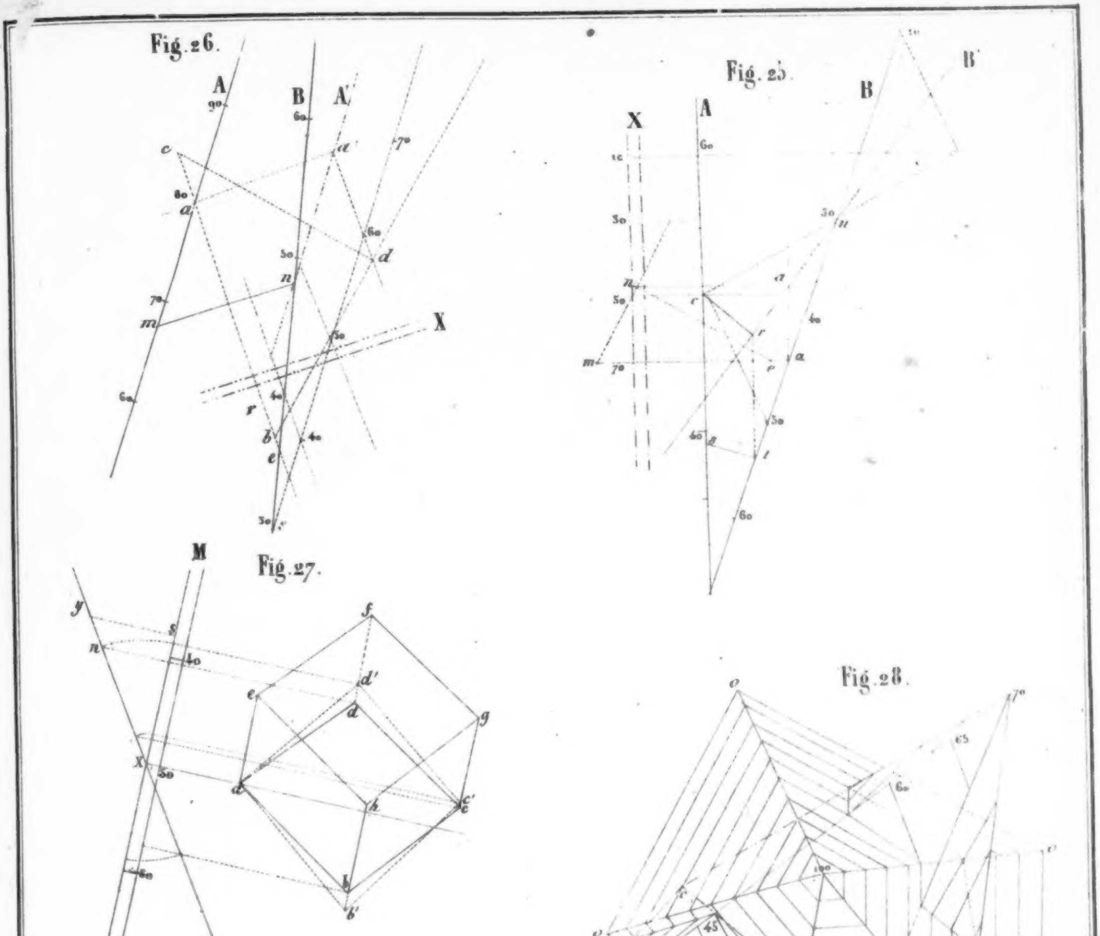
Fig. 26.
Fig. 25.
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Fig. 28.
Fig. 29.

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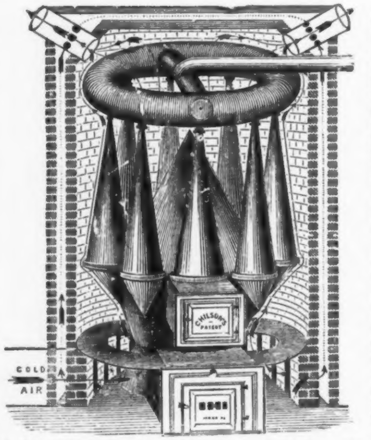

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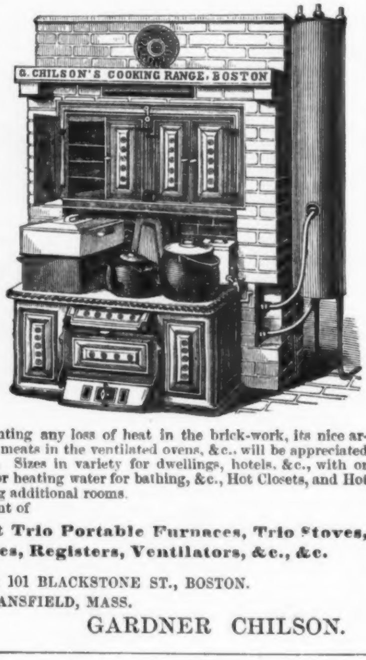

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